EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

European Journal of Operational Research 118 (1999) 450–463

www.elsevier.com/locate/orms

Theory and Methodology

The ability to compensate for suboptimal capacity decisions by optimal pricing decisions

Bernd Skiera *, Martin Spann

Department of Electronic Commerce, Goethe-University, Mertonstr. 17, 60054 Frankfurtam Main, Germany

Received 1 August 1997; accepted 1 September 1998

Abstract

Certain companies have high capacity cost and rather moderate production cost. These companies usually assume that deciding about their capacity is quite critical. Frequently, however, they are able to adjust the demand for their products to the available capacity by setting appropriate prices, that is higher (lower) than current prices in the presence of under-capacity (over-capacity). We argue that appropriate prices can reduce the adverse effects of non-optimal capacities. We analyze the sensitivity of profit in such a situation for a company in a monopolistic market, selling a non-storable product and facing fluctuating but interdependent demand across two time periods which allows to profitably differentiate prices. Therefore, we state optimality conditions for prices in situations of variable and given capacities and describe a procedure to determine them. The main suggestion of this analysis is that, within the bounds of the normative models and specific parameters examined, optimal prices can substantially reduce the adverse effects of capacity deviating from its optimum. In this way, profit is rather insensitive to deviations of capacity from its optimum. The implications of this finding are discussed for a number of situations.

Keywords: Marketing; Pricing; Capacity; Peak-load pricing; Flat maximum principle

1. Introduction

Companies which are, for example, organizing concerts, managing recreation centers or running hotels face the following problem. They have a limited capacity such as seats for a concert, tennis or squash courts for a recreation center and rooms in a hotel and want to sell the usage of that capacity in a way maximizing profit. Their situation is regularly characterized by at least some monopolistic power, a cost structure in which capacity cost – and hence fixed cost – are often predominant, and fluctuating but interdependent demand across time periods. Usually, such companies assume that choosing the optimal capacity is a rather critical decision (e.g. Greenley, 1989; Klinge, 1997). Frequently, however, they are able to adjust the demand for their products in each time period to the available capacity by increasing (decreasing) prices in the case that demand exceeds

* Corresponding author. Tel.: 49 69 798 22377; fax: 49 69 798 28973; e-mail: skiera@wiwi.uni-frankfurt.de

0377-2217/99/$ – see front matter
PII: S 0 3 7 7 - 2 2 1 7 (9 8) 0 0 3 4 6 - 4

(falls below) capacity. Hence, it seems possible to compensate for incurred losses in sales and profit by scarce capacities at least to some extent by higher prices, and to compensate for the additional cost for over-capacity by higher sales from lower prices.

Surprisingly, the ability to compensate for suboptimal capacity decisions by optimal pricing decisions has not been systematically analyzed in the literature. The primary focus of most papers dealing with the well-known peak-load pricing problem is on simultaneously optimizing the prices for the different time periods and the required capacity (for an overview see Crew et al. (1995)). The major part of these papers considers the case of welfare maximization in a situation of independent demand across time periods. Only few of them address profit maximization (Bailey and White, 1974; Burness and Patrick, 1991; Crew and Kleindorfer, 1986; Crew et al., 1995) and interdependent demand across time periods (Pressman, 1970; Burness and Patrick, 1991; Berg and Tschirhart, 1988; Crew and Kleindorfer, 1986). Yet, these profit maximization approaches regard the simultaneous optimization of prices and capacity only. The effect of a given (and not necessarily optimal) capacity on profit is only considered in welfare maximization approaches (Pressman, 1970; Berg and Tschirhart, 1988). However, apart from the statement of Berg and Tschirhart (1988) that welfare decreases if capacity deviates from the optimum, these approaches have not focused on determining the sensitivity of welfare to deviations of capacity from its optimum. The only ones who analyzed the effects of deviations of capacity from its optimum on welfare are Koschat et al. (1995). Their implication is that welfare is very sensitive to a varying capacity. Unfortunately, they only considered welfare (and not profit) and additionally held prices for different capacities constant, thus preventing prices from compensating for the adverse effects of a suboptimal capacity.

The aim of this paper is to analyze the ability to compensate for suboptimal capacity decisions by optimal pricing decision which could also be considered as the sensitivity of profit to varying sizes of capacity. We analyze this effect analytically and, where unavoidable, numerically in a static environment for a monopolistic firm which faces fluctuating but interdependent demand across two time periods. Therefore, we state optimality conditions for prices in situations of variable and given capacities and describe a procedure to determine them. The effects of different shapes of demand functions, varying relations between marginal production and capacity costs and different price as well as cross price elasticities for the different time periods on the sensitivity of profit are analyzed. The implications of our findings are discussed for a number of situations.

Our analysis contributes to the knowledge that has been derived by examining the validity of the "flat maximum principle" (Chintagunta, 1993; Silver and Tull, 1986; Tull et al., 1986). The studies within that area analyze the important question of whether it is really necessary to strive for optimal solutions or whether it suffices to aim for solutions which are within a fairly broad range (e.g. ±25%) around the optimum. Tull et al. (1986) and Chintagunta (1993) analyzed the sensitivity of profits for varying advertising budgets. Their result is that profit is rather insensitive as long as advertising elasticities are positive but smaller than approximately 0.5. This is the range usually found in empirical studies (see Assmus et al., 1984; Hanssens et al., 1990; Lodish et al., 1995). Although Tull et al. (1986) and Chintagunta (1993) focused only on the marketing instrument advertising, it is likely that the validity of the flat maximum principle for advertising budgets holds for other marketing budget instruments (e.g. sales force expenditures) as well. Silver and Tull (1986) analyzed the validity of the flat maximum principle for price as a marketing instrument. Their conclusion is that profit is insensitive as long as fixed cost is rather small. Thus, their analysis suggests that companies with high fixed cost (e.g. capacity cost) should carefully choose their prices.

In contrast to these studies, we analyze the sensitivity of profit to deviations of capacity from its optimum under the condition that optimal pricing decisions are used to compensate for the losses incurred by suboptimal capacity decisions. Hence, we are the first to analyze the validity of the flat maximum principle for decisions which are not typically within the responsibility of marketing managers, but whose suboptimality might be compensated for by marketing managers' good decisions. Thus, we analyze how strong the performances of marketing managers are affected by the capacity decisions usually made by others. Stated differently, we focus whether marketing managers in a situation of a suboptimal capacity are still able to act nearly as profitable as if they could decide about the available capacity. Consequently, we analyze whether complaints from marketing managers about substantial losses in profit due to suboptimal capacities are justified or not.

The remainder of this paper is organized as follows. In Section 2, we analyze the ability to compensate for suboptimal capacity decisions by optimal pricing decisions in the case where a company does not face a fluctuating demand across time periods, and hence, price differentiation across periods is not profitable. Section 3 extends this analysis to the case of varying and interdependent demand across two time periods that allows the company to profitably differentiate prices across periods (e.g. peak and off-peak prices). In each of these sections, we, first, outline the optimality conditions for prices in situations of variable and given capacities, second, describe a procedure to determine optimal prices and capacity, and, third, analyze the sensitivity of profit to deviations of capacity from its optimum either analytically or numerically. Implications and possible directions for future research in Section 4 conclude this contribution.

2. Analysis for uniform demand across time periods

In this section, we analyze the ability to compensate for suboptimal capacity decisions by optimal pricing decisions in case of an uniform demand across time periods for a profit maximizing company in a monopolistic market selling a non-storable product (such as the use of squash courts in a recreation center or rooms in a hotel). The cost structure is such that most of the costs are capacity cost and hence, marginal production cost is relatively low. We proceed as follows. First, we outline the optimality conditions for prices in situations of variable and given capacities and then describe a procedure to determine them. Next, we determine analytically the sensitivity of profit to deviations of capacity from its optimum. A closed-form solution of this deviation cannot be provided for general shapes of demand functions. Therefore, we choose the two most prominent demand functions (linear and multiplicative) and provide closed-form solutions as well as numerical examples for the sensitivity of profit for both of them. For the sake of simplicity, we assume constant marginal production and capacity cost throughout our analysis.

2.1. Optimal prices

We assume that a company's profit function can be stated in the following general form, where total costs consist of production and capacity costs which increase with the produced quantity and the capacity size, respectively. The demand and cost function are deterministic and continuously differentiable.

$$\pi = p \cdot x(p) - C(x(p), Q), \tag{1}$$

where π is the profit, $p > 0$ the price, $x(p)$ the price response function with $\partial x/\partial p < 0$, $C(x, Q)$ the cost function, $x > 0$ the total quantity produced and sold (non-storable good), $Q > 0$ the capacity, $\partial C/\partial x > 0$ the marginal production cost, $\partial C/\partial Q > 0$ the marginal capacity cost.

2.1.1. Variable capacity

If a company can adjust its production output as well as its capacity, then the solution of the following optimization problem maximizes the profit incurred:

$$\max_{P,Q} \pi = p\, x(p) - C(x(p), Q)$$
$$\text{subject to } x \leqslant Q. \tag{2}$$

Solving the corresponding Lagrange function leads to the following optimal price and capacity (see Simon, 1989; Varian, 1992):

$$p^* = \frac{\partial C/\partial x + \partial C/\partial Q}{1 + 1/\varepsilon} \quad \text{with } \varepsilon = \frac{\partial x}{\partial p}\frac{p}{x} < -1, \tag{3}$$

$$Q^* = x(p^*), \tag{4}$$

where p^* is the optimal price, $\varepsilon < -1$ the elasticity of demand with respect to price, Q^* the optimal capacity.

Eq. (3) is the well-known Amoroso–Robinson relation which states that the optimal price is achieved if marginal revenue equals marginal cost. The marginal cost term consists of marginal production cost *plus* marginal capacity cost. As we have assumed that marginal capacity cost exceeds marginal production cost, the optimal price is mostly influenced by the former. In addition, the optimal capacity equals the demand at the optimal price (Eq. (4)).

2.1.2. Given capacity

In many cases, however, companies cannot adjust their current (suboptimal) capacity on a short-run basis, thus capacity cost is fixed. In such a situation, the profit function (1) yields the Lagrange function (5):

$$L(p,\lambda) = p\,x(p) - C(x(p),\overline{Q}) + \lambda\,(\overline{Q} - x(p)). \tag{5}$$

The optimization of that Lagrange function leads to the following first order conditions: [1]

$$p^+ = \frac{\partial C/\partial x + \lambda}{1 + 1/\varepsilon}, \tag{6}$$

$$\overline{Q} - x \geqslant 0, \quad \lambda(\overline{Q} - x) = 0, \lambda \geqslant 0, \tag{7}$$

and the Lagrange multiplier:

$$\lambda = p^+(1 + 1/\varepsilon) - \partial C/\partial x, \tag{8}$$

where $\overline{Q}$ is the given capacity, p^+ the optimal price for the given capacity, λ the Lagrange multiplier.

The Lagrange multiplier λ in Eq. (6) replaces the marginal capacity cost in Eq. (3). The value of that Lagrange multiplier λ reflects the deviation of the capacity from its optimum. If the capacity is too small (large), the Lagrange multiplier λ will be higher (lower) than the marginal capacity cost, thus yielding higher (lower) prices. If the given capacity turns out to be optimal, λ equals the marginal capacity cost and p^+ equals p^*.

According to the Kuhn–Tucker conditions in Eq. (7) (see Pressman, 1970; Takayama, 1994), two different situations are possible for the optimal solution:

(i) The quantity sold equals the given capacity ($x = \overline{Q}$). Hence, the value of the Lagrange multiplier is either positive or equal to zero ($\lambda \geqslant 0$).

(ii) The quantity sold is smaller than the given capacity ($x < \overline{Q}$). Hence, the value of the Lagrange multiplier must be equal to zero ($\lambda = 0$) for the Kuhn–Tucker conditions to hold.

Thus, the value of the Lagrange multiplier λ in Eq. (8) can be used in order to determine which situation is the optimal one. Therefore, the following sequential procedure checks in a first step whether the optimum is described by a situation in which the quantity sold equals the given capacity, thus

$$\overline{Q} = x(p'), \tag{9}$$

where p' is the price in the presence of a given capacity that achieves a demand equal to capacity, such that the price p' is determined by the inverse price response function:

$$p' = x^{-1}[\overline{Q}] = x^{-1}[\tau Q^*] = x^{-1}[\tau\, x(p^*)], \tag{10}$$

where $p = x^{-1}[x]$ is the inverse price response function, $\tau = \overline{Q}/Q^*$ the relative deviation of the given capacity compared to the optimal capacity.

Eq. (10) yields the optimal price, if the Kuhn–Tucker conditions hold, i.e., if the Lagrange multiplier λ in Eq. (8) for the price p' is non-negative. Otherwise the second situation with the quantity sold below the given capacity is the optimal one. In this case, the Lagrange multiplier λ is equal to zero and the optimal price can be determined by Eq. (6).

2.2. Sensitivity analysis

In this section we derive analytically the sensitivity of profit to deviations of capacity from its optimum, and thus, the ability to compensate for

[1] Details of these and all other necessary calculations are available from the authors.

suboptimal capacity decisions by optimal pricing decisions. Therefore, we compare the profit of a company which is able to optimize its price and capacity (Eqs. (3) and (4)) with the profit of a company which has a given capacity (which might be suboptimal and, thus, deviate from the optimum) and, hence, can only optimize its price. To derive at closed-form solutions, we derive all results for the case that a company always sets the price in such a way that the quantity sold equals the given capacity. That way we only heuristically determine the price p' for the case of a given capacity. This price p' equals the *optimal* price p^+ if the Lagrange multiplier λ (Eq. (8)) is non-negative. Otherwise, profit could be increased by increasing the price to p^+, thus leaving part of the given capacity unused. Hence, the profit deviation δ' derived by comparing the results from simultaneously optimizing price and capacity by Eqs. (3) and (4) with the price p' set heuristically by Eq. (10) represents an *upper bound* for the sensitivity of profit to deviations of capacity from its optimum.

$$\delta' = \frac{\pi' - \pi^*}{\pi^*}, \tag{11}$$

where π^* is the profit for simultaneously optimizing price and capacity, π' the profit for a given capacity ($\overline{Q} = \tau Q^*$) and setting the price such that demand equals capacity.

In order to determine this profit deviation δ', we use Eqs. (3) and (4) to calculate p^* and Q^*, and thus π^*, and Eq. (10) to calculate p' and the information about the given capacity $\overline{Q} = \tau Q^*$ to determine π'. Inserting both profits π^* and π' in Eq. (11) and simplifying yields the following profit deviation:

$$\begin{aligned}\delta' &= \frac{\tau(p' - p^*)}{p^* - \partial C/\partial x - \partial C/\partial Q} + (\tau - 1) \\ &= \frac{\tau(x^{-1}[\tau x(p^*)] - p^*)}{p^* - \partial C/\partial x - \partial C/\partial Q} + (\tau - 1).\end{aligned} \tag{12}$$

Further simplification of Eq. (12) is only possible for specific price response functions. The meta-analyses from Tellis (1988) and Mauerer (1995) show that the linear and the multiplicative price response functions are the ones most often used. Therefore, we present the sensitivity of profit in a greater detail for these two response functions.

2.2.1. Multiplicative price response function

The main feature of a multiplicative price response function is a constant price elasticity of demand

$$x(p) = \alpha\, p^{\varepsilon}, \tag{13}$$

where $\alpha > 0$ is the scaling parameter.

Due to its iso-elasticity, this function implies a formulation of p^* which is identical to Eq. (3) for the case of optimizing price and capacity. For the case of a given capacity, we insert the inverse multiplicative price response function in Eq. (10), yielding

$$p' = \tau^{1/\varepsilon} \cdot p^* = \tau^{1/\varepsilon} \cdot \left(\frac{\partial C/\partial x + \partial C/\partial Q}{1 + 1/\varepsilon} \right). \tag{14}$$

Eq. (14) states that the price p' increases in case of a shortage of capacity ($\tau < 1$) (and vice versa) and that the relation of the two prices p^* and p' is only influenced by the deviation of capacity from its optimum and the price elasticity. For the profit deviation we yield

$$\delta' = \varepsilon\tau - \varepsilon\tau^{1/\varepsilon + 1} + \tau - 1. \tag{15}$$

Eq. (15) states that the profit deviation depends only on the deviation of capacity from its optimum and the price elasticity of demand. This profit deviation increases with higher absolute values for both of them.

In order to provide some numerical examples, we vary the price elasticity around the average values of -1.76 and -1.88 of the meta-analyses of Tellis (1988) and Mauerer (1995) in an interval of $[-1.1; -4]$. This range of values also corresponds approximately to the results of various empirical studies summarized in Hanssens et al. (1990). The capacity deviations are varied by up to $\pm 50\%$ around the optimum. For these variations, the price p' (Eq. (10)) is always optimal as long as marginal capacity costs are not smaller than marginal production costs.

Table 1
Profit deviation for varying price elasticities and capacities in the case of a multiplicative price response function.

δ' [%]	τ						
ε	0.5	0.75	0.9	Q^*	1.1	1.25	1.5
−1.10	−1.7	−0.3	0.0	0.0	0.0	−0.2	−0.9
−1.25	−3.7	−0.7	−0.1	0.0	−0.1	−0.5	−1.9
−1.50	−5.9	−1.2	−0.2	0.0	−0.2	−0.9	−3.3
−1.75	−7.5	−1.5	−0.2	0.0	−0.2	−1.2	−4.3
−2.00	−8.6	−1.8	−0.3	0.0	−0.2	−1.4	−5.1
−2.25	−9.4	−2.0	−0.3	0.0	−0.3	−1.6	−5.7
−2.50	−10.1	−2.1	−0.3	0.0	−0.3	−1.7	−6.1
−2.75	−10.6	−2.3	−0.3	0.0	−0.3	−1.8	−6.5
−3.00	−11.0	−2.4	−0.3	0.0	−0.3	−1.9	−6.9
−3.25	−11.4	−2.4	−0.4	0.0	−0.3	−2.0	−7.2
−3.50	−11.7	−2.5	−0.4	0.0	−0.3	−2.0	−7.4
−3.75	−11.9	−2.6	−0.4	0.0	−0.4	−2.1	−7.6
−4.00	−12.2	−2.6	−0.4	0.0	−0.4	−2.1	−7.8

Table 1 summarizes the profit deviations for these variations. The profit deviation increases with higher deviations of capacity from its optimum and higher absolute values of the elasticities. Yet, these results indicate that profit is rather insensitive to deviations of capacity from its optimum. A price elasticity of −2 and a given capacity of 25% below (above) the optimal capacity, for example, reduces profit only by 1.8% (1.4%). It is worth noting that the profit is less sensitive to an over-sized capacity than to an under-sized one. This is due to the non-linear effect of a multiplicative price response function that the absolute sales effect of a price change increases with lower prices (see Simon, 1989).

2.2.2. Linear price response function

The second function which we analyze in more detail is the linear price response function

$$x(p) = a - mp \text{ with } \varepsilon = \frac{-mp}{a - mp} < 0 \quad \text{for } p < a/m, \tag{16}$$

where $a > 0$ is the maximum of sales, $m > 0$ the absolute change in sales resulting from a change in price by one unit, a/m the maximum price (see Simon, 1989).

In this function, the value of the price elasticity of demand changes with price. Eqs. (17) and (4) determine the optimal price and capacity.

$$p^* = \frac{(\partial C/\partial x + \partial C/\partial Q)m + a}{2m}. \tag{17}$$

The heuristic that the demand equals the given capacity yields in Eq. (18) p'.

$$p' = \frac{2a + \tau((\partial C/\partial x + \partial C/\partial Q)m - a)}{2m}. \tag{18}$$

Numerical analyses showed that this price p' (Eq. (18)) leads more often to non-optimal prices than the corresponding price for the multiplicative price response function in Eq. (14) and, thus, leaves, most probably, more room for improvement.

The substitution of Eqs. (17) and (18) in Eq. (12) results in the following – surprisingly simple – formula for the profit deviation:

$$\delta' = -(\tau - 1)^2. \tag{19}$$

That formula (19) states that the profit deviation depends *only* on the deviation of capacity from its optimum. Numerical examples for various deviations of capacity from its optimum are shown in Table 2. These results indicate that the profit deviation is higher for a linear than for a multiplicative price response function. However, for a given capacity in a range of ±25% around the optimal capacity, profit decreases only by 6.3%. Tull et al. (1986) still consider this sensitivity of profit as "flat". The sensitivity is symmetric around the optimum (see Fig. 1) because of the

Table 2
Profit deviations for varying capacities in case of a linear price response function

τ	0.5	0.75	0.9	Q^*	1.1	1.25	1.5
δ' (%)	−25.0	−6.3	−1.0	0.0	−1.0	−6.3	−25.0

linear price effect that a linear price response function has on the demand.

Fig. 1 compares the plots of the profit deviation for the two analyzed price response functions. This figure exhibits an extremely flat maximum for multiplicative price response functions, even for price elasticities as high as −3.5. The profit deviation for linear response functions is substantially higher. Yet, the effects of deviations of capacity of up to ±25% from its optimum are not higher than −6.3%. Therefore, the sensitivity of profit is low even in the case of a linear price response function. In cases where the price p' is not optimal, the sensitivity of profit might be decreased by using the optimal price p^+ instead of the price p' which has been derived heuristically. Due to these results, we conclude that the ability to compensate for suboptimal capacity decisions by optimal pricing decisions is high in case of an uniform demand across time periods.

3. Analysis for varying and interdependent demand across time periods

In this section, we extend our analysis to the case of varying and interdependent demand across two equal-length time periods where it is profitable for companies to differentiate prices across time periods (e.g. peak and off-peak prices). We proceed with our analysis in a similar way to the proceeding section. First, we outline the optimality conditions for prices in situations of variable and given capacities and describe a procedure to determine them. Then, we analyze numerically and partly analytically the sensitivity of profit to deviations of capacity from its optimum. Again, as we cannot determine the sensitivity of profit for general shapes of demand functions, we use multiplicative and linear price response functions to provide a more detailed analysis.

3.1. Optimal prices

For the case of varying and interdependent demand across time periods, we expand the profit function in (1) to (20) and assume, as frequently has been done in the literature (e.g. Crew et al., 1995), that the demand of the peak period lies everywhere above the demand of the off-peak period for similar prices and that the two periods are of equal length:

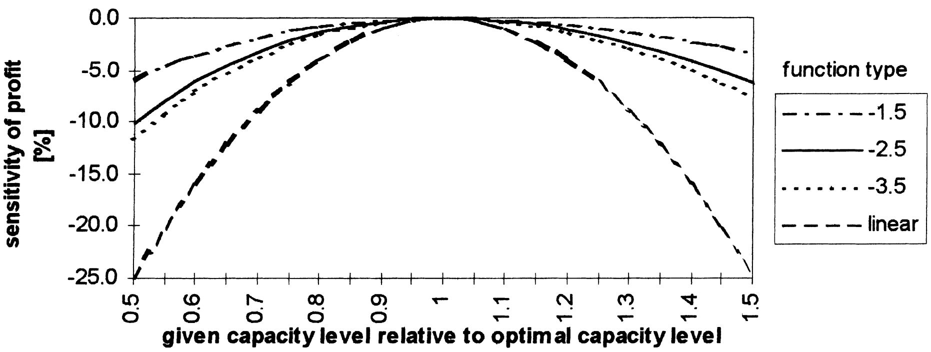

Fig. 1. Sensitivity of profit for different shapes of demand functions (multiplicative functions are labeled with the value of their elasticity, e.g. −2.5).

$$\pi_T = p_1x_1(p_1,p_2) + p_2x_2(p_1,p_2) - C(x_1(p_1,p_2),x_2(p_1,p_2),Q), \qquad (20)$$

where subscript 1 denotes the peak period, subscript 2 denotes the off-peak period, subscript T denotes the case of fluctuating demand across time periods, π_T is the profit of both periods, $p_{1(2)} > 0$ the price in the peak (off-peak) period, $x_{1(2)}(p_1,p_2)$ the price response function of the peak (off-peak) period, $C(x_1,x_2,Q)$ the cost function, $x_{1(2)} > 0$ the total quantity produced and sold (non-storable good) in the peak (off-peak) period, $\partial C/\partial x_{1(2)} > 0$ the marginal production cost in the peak (off-peak) period.

3.1.1. Variable capacity

If a company is able to optimize prices in each of the two periods as well as capacity, the following maximization problem needs to be solved:

$$\max_{p_1,p_2,Q} \quad \pi_T = p_1x_1(p_1,p_2) + p_2x_2(p_1,p_2) - C(x_1(p_1,p_2),x_2(p_1,p_2),Q)$$
$$\text{s.t. } x_1 \leqslant Q,\ x_2 \leqslant Q. \qquad (21)$$

The corresponding Lagrange function yields the following solution:

$$\frac{p_1^* \cdot \left(\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2 + \varepsilon_2 - \frac{x_1}{x_2}\gamma_1\right) + p_2^* \cdot \left(\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2 + \varepsilon_1 - \frac{x_2}{x_1}\gamma_2\right)}{(\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2)} = \frac{\partial C}{\partial x_1} + \frac{\partial C}{\partial x_2} + \frac{\partial C}{\partial Q}, \qquad (22)$$

with

$$|\varepsilon_i| > |\gamma_j|, \quad \varepsilon_{1(2)} < -1,$$

and

$$\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2 > \max\left|\varepsilon_i - (x_j/x_i)\gamma_j\right| \quad \text{for } i,j = 1,2 \text{ and } i \neq j,$$

$$Q^* - x_1 \geqslant 0, \quad \lambda_1(Q^* - x_1) = 0, \quad \lambda_1 \geqslant 0, \qquad (23)$$

$$Q^* - x_2 \geqslant 0, \quad \lambda_2(Q^* - x_2) = 0, \quad \lambda_2 \geqslant 0, \qquad (24)$$

where

$\varepsilon_{1(2)} = \left(p_{1(2)}/x_{1(2)}\right)\left(\partial x_{1(2)}/\partial P_{1(2)}\right)$ is the elasticity of the demand of the peak (off-eak) period with respect to the price of the peak (off-peak) period (i.e. the direct price elasticity),

$\gamma_{2(1)} = \left(p_{2(1)}/x_{1(2)}\right)\left(\partial x_{1(2)}/\partial P_{2(1)}\right)$ is the elasticity of the demand of the peak (off-peak) period with respect to the price of the off-peak (peak) period (i.e. the cross-price elasticity),

$p^*_{1(2)}$ is the optimal price of the peak (off-peak) period.

Eq. (22) represents a specification of the well-known "marginal revenue equals marginal cost"-condition for monopolistic profit maximization (e.g. Simon, 1989; Varian, 1992). Therefore, the profit maximizing price structure is characterized by the equality of joint marginal revenues (left side of Eq. (22)) and joint marginal costs (right side of Eq. (22)) in both periods. At least the peak demand is always equal to capacity because capacity cost strictly increases with capacity and the assumption that the peak demand is always in excess of the off-peak demand for similar prices in the two periods. Therefore, the procedure to calculate the optimal solution has to consider the two possible situations that:

(i) either the demand in both periods equals the optimal capacity, or

(ii) the peak demand equals the optimal capacity and the off-peak demand is smaller than the optimal capacity.

Since peak demand is always equal to optimal capacity, the corresponding Kuhn–Tucker conditions (Eq. (23)) hold. Therefore, optimal prices and capacity are again determined by a sequential procedure which is build upon the Kuhn–Tucker conditions for the off-peak period (Eq. (24)).

In a first step, the solution for the optimal prices and the optimal capacity is derived for the situation that both demands are equal to the capacity

$$Q^* = x_1\left(p_1^*,p_2^*\right) = x_2\left(p_1^*,p_2^*\right). \qquad (25)$$

Eqs. (22) and (25) are used to determine the optimal prices p_1^*,p_2^* and the optimal capacity Q^*. According to Eq. (24), this result is an optimum if the Lagrange multiplier of the off-peak period λ_2 (Eq. (26)) is non-negative.

$$\lambda_2 = p_2^* + \frac{p_2^*\varepsilon_1 - p_1^*(x_1/x_2)\gamma_1}{\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2} - \frac{\partial C}{\partial x_2}. \tag{26}$$

Otherwise the optimal solution is characterized by an off-peak demand below optimal capacity such that λ_2 equals zero

$$Q^* = x_1(p_1^*, p_2^*) > x_2(p_1^*, p_2^*). \tag{27}$$

Thus, the maximization problem in Eq. (21) has to be solved under the condition that only the peak demand equals the capacity which leads to the following solution:

$$p_1^* = \frac{\varepsilon_1}{1+\varepsilon_1}\left(\frac{\partial C}{\partial x_1} + \frac{\partial C}{\partial Q}\right) - \frac{\frac{x_2}{x_1}\gamma_2(p_2^* - \partial C/\partial x_2)}{1+\varepsilon_1}, \tag{28}$$

$$p_2^* = \frac{\varepsilon_2}{1+\varepsilon_2}\frac{\partial C}{\partial x_2} - \frac{x_1/x_2\,\gamma_1[p_1^* - (\partial C/\partial x_1 + \partial C/\partial Q)]}{1+\varepsilon_2}. \tag{29}$$

The optimal prices and the optimal capacity are calculated by Eqs. (27)–(29). Now, only the peak demand completely uses and accordingly determines the capacity (Eq. (27)). Therefore, the peak price has to bear the whole marginal capacity cost (Eqs. (28) and (29)). Thus, in contrast to the former decision rule in (22), marginal capacity cost is no longer split between the prices of the two periods.

3.1.2. Given capacity

In this case, the profit function (20) has to be modified to incorporate the capacity constraints for the peak and off-peak demand ($x_1 \leqslant \overline{Q}$ and $x_2 \leqslant \overline{Q}$). That modified profit function yields the following Lagrange function:

$$\begin{aligned} L(p_1, p_2, \lambda_1, \lambda_2) = {} & p_1x_1(p_1,p_2) + p_2x_2(p_1,p_2) \\ & - C(x_1(p_1,p_2), x_2(p_1,p_2), \overline{Q}) + \lambda_1(\overline{Q} \\ & - x_1(p_1,p_2)) + \lambda_2(\overline{Q} - x_2(p_1,p_2)). \end{aligned} \tag{30}$$

The optimization of the Lagrange function (30) leads to the following first order conditions:

$$\begin{aligned} p_{1(2)} = {} & \frac{\varepsilon_{1(2)}}{1+\varepsilon_{1(2)}}\left(\frac{\partial C}{\partial x_{1(2)}} + \lambda_{1(2)}\right) \\ & - \left[p_{2(1)} - \left(\frac{\partial C}{\partial x_{2(1)}} + \lambda_{2(1)}\right)\right]\frac{(x_{2(1)}/x_{1(2)})\gamma_{2(1)}}{(1+\varepsilon_{1(2)})}, \end{aligned} \tag{31}$$

$$\overline{Q} - x_{1(2)} \geqslant 0, \quad \lambda_{1(2)}(\overline{Q} - x_{1(2)}) = 0, \quad \lambda_{1(2)} \geqslant 0, \tag{32}$$

and the Lagrange multipliers

$$\begin{aligned} \lambda_{1(2)} = {} & p_{1(2)} \\ & + \frac{p_{1(2)}\varepsilon_{1(2)} - p_{2(1)}(x_{2(1)}/x_{1(2)})\gamma_{2(1)}}{\varepsilon_1\varepsilon_2 - \gamma_1\gamma_2} - \frac{\partial C}{\partial x_{1(2)}}, \end{aligned} \tag{33}$$

where $p_{1(2)}^+$ is the optimal price in the peak (off-peak) period for a given capacity, $\lambda_{1(2)}$ the Lagrange multiplier of the peak (off-peak) period.

Comparable to the case of a uniform demand, the marginal capacity costs in Eq. (22) are substituted by the Lagrange multipliers of both periods (Eq. (31)). According to Eq. (33), the Lagrange multipliers represent marginal revenues less marginal production cost, i.e. the marginal profit at the capacity limit.

Now, the capacity cost represents fix cost such that three different situations are possible for the optimal solution (see also the Kuhn–Tucker conditions in Eq. (32)):

(i) The demand in both periods equals the given capacity ($\overline{Q} = x_1 = x_2$). Accordingly, the values of the Lagrange multipliers of both periods are either positive or equal to zero ($\lambda_1 \geqslant 0$, $\lambda_2 \geqslant 0$).

(ii) The peak demand equals the given capacity and the off-peak demand is smaller than the given capacity ($\overline{Q} = x_1 > x_2$). Hence, the Lagrange multiplier of the peak period is either positive or equal to zero and the Lagrange multiplier of the off-peak period must be equal to zero ($\lambda_1 \geqslant 0$, $\lambda_2 = 0$).

(iii) Peak and off-peak demand are smaller than the given capacity. As a result of Eq. (32), the Lagrange multipliers of both periods must be equal to zero ($\lambda_1 = 0, \lambda_2 = 0$).

Again, the optimal solution is derived through a sequential three-step procedure which examines first the situation in which the demands of both periods equal the given capacity:

$$\overline{Q} = x_1(p_1^+, p_2^+), \tag{34}$$

$$\overline{Q} = x_2(p_1^+, p_2^+). \tag{35}$$

The solution for the first situation (i.e. the variables p_1^+, p_2^+, λ_1 and λ_2) is determined by the two Eq. (31) for the prices p_1^+, p_2^+, and Eqs. (34) and (35). Accordingly to the Kuhn–Tucker conditions in Eq. (32), this solution is the optimum if the Lagrange multipliers of the peak- and the off-peak period (Eq. (33)) are non-negative.

Otherwise, the second situation is examined in which peak demand is equal and off-peak demand is smaller than the given capacity. Hence, the Lagrange multiplier of the off-peak period must be equal to zero for the Kuhn–Tucker conditions of the off-peak period to hold. The solution of the second situation (i.e. the variables p_1^+, p_2^+, and λ_1) is determined by the two equations (31) for the prices p_1^+, p_2^+, and Eq. (34). This solution is optimal if the Lagrange multiplier of the peak period is non-negative.

Otherwise, the third situation with both demands smaller than the given capacity is the optimal solution. In this case, the Lagrange multipliers must be equal to zero and the optimal solution is determined (i.e. the variables p_1^+, p_2^+,) by the two equations (31) for the prices p_1^+, p_2^+.

3.2. Sensitivity analysis

In this section we analyze numerically and partly analytically the sensitivity of profit to deviations of capacity from its optimum, and thus, the ability to compensate for suboptimal capacity decisions by optimal pricing decisions, in situations of varying and interdependent demand across two time periods of equal-length. Again, we compare the profit for the optimization of prices and capacity with the profit in the situation where only prices are optimized for a given capacity. Thus, the profit deviation is stated in the following form:

$$\delta_T^+ = \frac{\pi_T^+ - \pi_T^*}{\pi_T^*} \tag{36}$$

where δ_T^+ is the profit deviation applying optimal prices (p_1^*, p_2^*, p_1^+ and p_2^+,), π_T^* the profit for simultaneously optimizing prices and capacity, π_T^+ the profit for a given capacity and setting the optimal prices p_1^+, p_2^+.

Again, simplification is only possible if price response functions are stated in a specific form. Thus, we continue to present a more detailed analysis for multiplicative and linear price response functions in both periods.

3.2.1. Multiplicative price response function

All price elasticities of multiplicative price response functions are constant. Since we have substitution effects between both periods, the functions possess negative direct and positive cross price elasticities:

$$x_1(p_1, p_2) = \alpha_1 p_1^{\varepsilon_1} p_2^{\gamma_1}, \tag{37}$$

$$x_2(p_1, p_2) = \alpha_2 p_2^{\varepsilon_2} p_1^{\gamma_2}, \tag{38}$$

with $|\varepsilon_i| > |\gamma_j|$ for $i \neq j, \varepsilon_{1(2)} < -1$ and $\gamma_{1(2)} > 0$, where $\alpha_{1(2)} > 0$ is the scaling parameter of the peak (off-peak) period.

The numerical analysis in the case where only the peak demand is equal to the optimal capacity always yields an optimal off-peak price converging to infinity and an optimal peak price converging to the value of the Amoroso–Robinson relation for a uniform demand (Eq. (3)). Hence, if the off-peak demand is relatively small compared to that of the peak period and if the substitution effect of the off-peak price on the peak demand is rather strong, it is optimal to refrain from off-peak sales in favor of peak sales. This result occurs because the elasticities and hence the price-effects of multiplicative functions are constant, even if prices converge on infinity. Our result of an off-peak price converging to infinity implies an offering in the off-peak period to be unfavorable and thus leads to a problem of uniform demand that has been analyzed in Section 2.2.1.

For our numerical examples, we choose values for the rather price-insensitive peak (−1.2; −1.8), the more price-sensitive off-peak period (−2; −4) as well as cross price elasticities (0.2; 0.6) which correspond with the values reported in the meta-analyses of Tellis (1988) and Mauerer (1995). We analyze all of the $2^4 = 16$ possible combinations. Further, we assume identical scaling parameters and a cost structure consisting predominantly of capacity cost: $\partial C/\partial x_1 = \partial C/\partial x_2 = 2.5$ and $\partial C/\partial Q = 20$. Table 3 shows the profit deviation for these values of price elasticities and capacity deviations of up to ±50% from its optimum. Again we obtain a low sensitivity of profit for multiplicative price response functions.

3.2.2. Linear price response function

The single-price linear price response function (Eq. (16)) is extended to the case of substitutive price interdependencies:

$$x_1 = a_1 - m_1 p_1 + n_1 p_2 \quad (\text{for } m_{1(2)} > n_{2(1)}) \quad \text{for the peak period,} \tag{39}$$

$$x_2 = a_2 - m_2 p_2 + n_2 p_1 \quad (\text{for } m_{1(2)} > n_{2(1)}) \quad \text{for the off-peak period,} \tag{40}$$

where $a_{1(2)} > 0$ is the maximum of sales for the peak (off-peak) period, $m_{1(2)} > 0$ the absolute change in sales of the peak (off-peak) period resulting from changing the peak (off-peak) price by one unit, $n_{1(2)} > 0$ the absolute change in sales of the peak (off-peak) period resulting from changing the off-peak (peak) price by one unit, $m_{1(2)} > n_{2(1)}$ the direct price effect stronger than cross price effect.

Table 4 presents the results for numerical analysis of the sensitivity of profit to deviations of capacity from its optimum for different values of the ratio $a_{1(2)}/m_{1(2)}$, of $n_{1(2)}$ and for deviations of capacity of up to 50% from its optimum. Again we analyze all of the $2^4 = 16$ possible combinations and an identical cost structure. The elasticities in all combinations are within the range of the values reported in the meta-analysis of Tellis (1988) and Mauerer (1995). The results in Table 4 show again a low sensitivity of profit because the profit deviation for a capacity of 25% above or below the optimum is only −6.3.

Additionally, the sensitivity of profit to deviations of capacity from its optimum can be analyzed analytically for the case that the simultaneous optimization of prices and capacity leads to a demand in both periods that equals capacity. Proceeding in a comparable way to that in Section 2.2.2 yields the same simple formula (41) for an upper bound of the sensitivity.

Table 3
Profit deviation for varying price elasticities and capacities in the case of a multiplicative price response function and fluctuating but interdependent demand across time periods (values in italics only the peak period is offered)

δ_T^+ (%)				τ						
ε_1	ε_2	γ_1	γ_2	0.5	0.75	0.9	Q^*	1.1	1.25	1.5
−1.8	*−4*	*0.6*	*0.6*	*−7.7*	*−1.6*	*−0.2*	*0.0*	*−0.2*	*−1.2*	*−4.5*
−1.8	*−4*	*0.6*	*0.2*	*−7.7*	*−1.6*	*−0.2*	*0.0*	*−0.2*	*−1.2*	*−4.5*
−1.8	−4	0.2	0.6	−7.5	−1.6	−0.2	0.0	−0.2	−1.2	−4.3
−1.8	−4	0.2	0.2	−7.5	−1.6	−0.2	0.0	−0.2	−1.2	−4.3
−1.8	−2	0.6	0.6	−4.1	−0.8	−0.1	0.0	−0.1	−0.6	−2.2
−1.8	*−2*	*0.6*	*0.2*	*−7.7*	*−1.6*	*−0.2*	*0.0*	*−0.2*	*−1.2*	*−4.5*
−1.8	−2	0.2	0.6	−5.9	−1.2	−0.2	0.0	−0.2	−0.9	−3.3
−1.8	−2	0.2	0.2	−7.1	−1.5	−0.2	0.0	−0.2	−1.1	−4.0
−1.2	*−4*	*0.6*	*0.6*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	*−4*	*0.6*	*0.2*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	*−4*	*0.2*	*0.6*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	*−4*	*0.2*	*0.2*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	*−2*	*0.6*	*0.6*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	*−2*	*0.6*	*0.2*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*
−1.2	−2	0.2	0.6	−1.4	−0.3	0.0	0.0	0.0	−0.2	−0.7
−1.2	*−2*	*0.2*	*0.2*	*−3.1*	*−0.6*	*−0.1*	*0.0*	*−0.1*	*−0.5*	*−1.6*

Table 4
Profit deviation for varying parameter values and capacities in the case of a linear price response function and fluctuating but interdependent demand across time periods (values in italics only the peak demand equals the optimal capacity; elasticities calculated at the optimal capacity level)

δ_T^+ (%)				τ										
a_1/m_1	a_2/m_2	n_1	n_2	0.5	0.75	0.9	Q^*	1.1	1.25	1.5	ε_1	ε_2	γ_1	γ_2
125	*15*	*10*	*10*	−24.3	−5.7	−0.8	*0.0*	−0.8	−4.5	−11.9	−1.5	−2.0	0.1	0.8
125	*15*	*10*	*5*	−23.2	−5.3	−0.8	*0.0*	−0.8	−5.0	−13.0	−1.5	−2.2	0.1	0.5
125	15	5	10	−25.0	−6.2	−1.0	0.0	−0.9	−5.0	−12.6	−1.5	−1.7	0.1	0.8
125	*15*	*5*	*5*	−24.1	−5.7	−0.8	*0.0*	−0.8	−5.0	−13.2	−1.5	−1.7	0.1	0.4
125	*25*	*10*	*10*	−24.7	−6.0	−0.9	*0.0*	−0.6	−3.7	−9.8	−1.6	−1.8	0.2	0.8
125	*25*	*10*	*5*	−22.6	−4.9	−0.8	*0.0*	−0.8	−4.4	−11.4	−1.5	−2.0	0.2	0.5
125	25	5	10	−25.0	−6.2	−1.0	0.0	−1.0	−4.7	−11.3	−1.6	−1.7	0.1	0.8
125	*25*	*5*	*5*	−24.2	−5.7	−0.8	*0.0*	−0.7	−4.4	−11.6	−1.5	−1.5	0.1	0.4
30	15	10	10	−25.0	−6.3	−1.0	0.0	−1.0	−6.3	−25.0	−3.5	−2.9	0.2	0.4
30	15	10	5	−25.0	−6.3	−1.0	0.0	−1.0	−6.3	−25.0	−3.7	−2.8	0.2	0.2
30	15	5	10	−25.0	−6.3	−1.0	0.0	−1.0	−6.3	−25.0	−3.7	−3.1	0.1	0.4
30	15	5	5	−25.0	−6.3	−1.0	0.0	−1.0	−6.2	−25.0	−4.0	−3.1	0.1	0.2
30	25	10	10	−25.0	−6.3	−1.0	0.0	−1.0	−6.2	−24.3	−3.0	−2.3	0.3	0.4
30	25	10	5	−25.0	−6.2	−1.0	0.0	−1.0	−6.2	−24.0	−3.1	−2.2	0.3	0.2
30	25	5	10	−25.0	−6.2	−1.0	0.0	−1.0	−6.2	−24.9	−2.9	−2.4	0.1	0.4
30	25	5	5	−25.0	−6.3	−1.0	0.0	−1.0	−6.3	−24.7	−3.2	−2.5	0.2	0.2

$$\delta'_T = -(\tau - 1)^2. \tag{41}$$

Again, the sensitivity of profit to deviations of capacity from its optimum is rather low.

4. Conclusions

Our analysis shows that the ability to compensate for suboptimal capacity decisions by optimal pricing decisions is fairly high so that profit is rather insensitive to a varying capacity. The reason for this insensitivity is that adjustments in prices can be used to compensate for the adverse effects of capacity deviations from the optimum. Hence, the main conclusion of this paper is that suboptimal capacities have only a minor influence on profits as long as prices are set optimally. This, in turn, infers two major implications. First, it suggests that complaints of marketing managers about substantial losses in profit being incurred by suboptimal capacity sizes are, at least for the normative models and parameters examined here, not justified. Rather, these managers might actually fail to set prices appropriately. Second, it indicates that managers who report about substantial increases in profit by augmenting their capacity have not, at least under the circumstances analyzed here, set prices in an optimal way before their adjustment of capacity. Hence, capacity problems are in fact pricing problems. Consequently, managers facing a demand higher (lower) than the available capacity should consider increasing (decreasing) the corresponding prices as a viable way out.

This insensitivity of profit accords companies a high flexibility in establishing their capacities. Thus, rules of thumb might be sufficient to decide about the capacity to be build up. However, pricing decision have to be made very carefully. That also indicates that managers should make sure that sufficient resources are available to support good pricing decisions. Decisions on capacity levels might also be guided by strategic considerations. When employing market penetration strategies, companies will not have to fear significant losses in profit from building up large capacities. On the other hand, companies being reluctant to build up large capacities, e.g. due to a high risk or a small financial budget, might opt for lower capacities without having to suffer substantial losses in profit.

Researchers who attempt to relate capacity size to profit in empirical studies should make sure that they really cover a broad range of capacity sizes and thus, most probably, large deviations of the capacities from their respective optimums. Otherwise, the demonstrated insensitivity would make it rather difficult to establish a significant relationship between these two variables. Researchers attempting to find a relationship between prices and profit might encounter similar difficulties if the varying prices arise from varying capacities.

A relaxation of the assumption of equal-length time periods usually displays the off-peak period to be relatively longer than the peak period. The portion of the off-peak period on total revenues usually increases with its relative duration. Since the peak demand lies everywhere above the off-peak demand, the effect of a suboptimal capacity on profit is dominated by the effect on the peak period. Hence, an increasing portion of the off-peak period on total revenues relatively diminishes the profit losses due to a suboptimal capacity during the peak period. If the profit-losses (due to a suboptimal capacity) are comparably smaller during the off-peak period, a shorter peak period will lead to an even lower sensitivity of profit than the assumption of equal-length time periods.

Future research might want to verify our results in empirical projects and investigate whether managers might have reasons not considered here to refrain from setting prices in the optimal way to adjust demand to their available capacity. Furthermore, it would be interesting to know whether our results hold for other situations than those considered here, too. Such situations might consider more than two time periods or other functional forms for the response functions.

Future research might also use our results and analyze their effects upon the regulation of monopolies. Lower prices increase consumer surplus and, as long as producer surplus is reduced by a lower amount, welfare. Therefore, we would suggest to require a regulated company to build up rather high capacities because high capacities concede lower prices. The impact of this high capacity on losses in profit of the regulated company should be fairly low whereas the lower prices should substantially increase consumer surplus. Hence, the result would be a gain in welfare that might be "almost Pareto-optimal".

Acknowledgements

We appreciated many helpful comments from Sönke Albers, Horst Herberg, Joachim Schleich and the three anonymous referees. Any remaining errors are the responsibility of the authors. Financial support provided by the Schmalenbach Gesellschaft is gratefully acknowledged.

References

Assmus, G., Farley, J.U., Lehmann, D.R., 1984. How advertising affects sales: Meta-analysis of econometric results. Journal of Marketing Research 21, 65–74.

Bailey, E.E., White, L.J., 1974. Reversals in peak and off-peak prices. Bell Journal of Economics and Management Science 5, 75–92.

Berg, S.V., Tschirhart, J., 1988. Natural Monopoly Regulation, Cambridge University Press, Cambridge.

Burness, H.S., Patrick, R.H., 1991. Peak-load pricing with continuous and interdependent demand. Journal of Regulatory Economics 3, 69–88.

Chintagunta, P.K., 1993. Investigating the sensitivity of equilibrium profit to advertising dynamics and competitive effects. Management Science 39, 1146–1162.

Crew, M.A., Kleindorfer, P.R., 1986. The Economics of Public Utility Regulation, Macmillan, London.

Crew, M.A., Fernando, C.S., Kleindorfer, P.R., 1995. The theory of peak-load pricing: A survey. Journal of Regulatory Economics 8, 215–248.

Greenley, G.E., 1989. Strategic Management, Prentice Hall International, Hempstead.

Hanssens, D.M., Parsons, L.J., Schultz, R.L., 1990. Market Response Models: Econometric and Time Series Analysis, Kluwer Academic Publishers, Dordrecht.

Klinge, R.C., 1997. Kapazitätsplanung in Dienstleistungsunternehmen: Planungs- und Gestaltungsprobleme, Gabler Verlag, Wiesbaden.

Koschat, M.A., Srinagesh, P., Uhler, L.J., 1995. Efficient price and capacity choices under uncertain demand: An empirical analysis. Journal of Regulatory Economics 7, 5–26.

Lodish, L.M., Abraham, M., Kalmenson, S., Livelsberger, J., Lubetkin, B., Richardson, B., Stevens, M.E., 1995. How TV advertising works: A meta-analysis of 389 real world split cable TV advertising experiments. Journal of Marketing Research 32, 125–139.

Mauerer, N., 1995. Die Wirkung absatzpolitischer Instrumente. Metaanalyse empirischer Forschungsarbeiten, Gabler Verlag, Wiesbaden.

Pressman, I., 1970. A mathematical formulation of the peak-load pricing problem. Bell Journal of Economics and Management Science 1, 304–326.

Silver, M., Tull, D.S., 1986. Pricing and the flat-maximum principle. Managerial and Decision Economics 7, 203–206.

Simon, H., 1989. Price Management, Elsevier, Amsterdam.
Takayama, A., 1994. Analytical Methods in Economics, Harvester Wheatsheaf, New York.
Tellis, G.J., 1988. The price elasticity of selective demand: A meta-analysis of econometric models of sales. Journal of Marketing Research 25, 331–341.
Tull, D.S., Wood, V.R., Duhan, D., Gillpatrick, T., Robertson, K.R., Helgeson, J.G., 1986. Leveraged decision making in advertising: The flat maximum principle and its implications. Journal of Marketing Research 23, 25–32.
Varian, H.R., 1992. Microeconomic Analysis, Norton, New York.